Name:________________________ Date:________________________

HW Math 12 Honors: Section 6.4 Polynomials with Complex Roots:

1. When given a polynomial in the form $y = x^4 + 3x^3 + 5x^2 + 12x + 4$, how many roots are there? If some of the roots are complex, what do you know about these complex roots?

2. Suppose you are given a polynomial $y = x^4 + Ax^2 + Bx + C$, where all the coefficients are real numbers. If one of roots is $z = 2 + 3i$ then find another root.

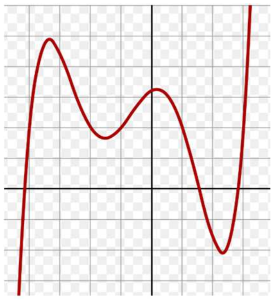
3. Given the polynomial $y = x^3 + x^2 + Bx + C$, where one root is $z = 1 + \sqrt{2}i$, what are the other two roots and what is the value of "C"?

4. Use synthetic division or long division to find the quotient and remainder.
$(x^4 + 16x^3 + 67x^2 + 63x - 70) \div (x + 10)$

5. How can you tell if a polynomial function $P(x) = ax^3 + bx^2 + cx + d$ is divisible by $(x - e)$? What does it mean if the polynomial is divisible by the binomial factor ? Explain:

6. Given the polynomial function, which of the following will give you the sum of all the coefficients?
$P(x) = x^n + Bx^{n-1} + Cx^{n-2} + Dx^{n-3} + ... + E$:
$i) P(1)$ ii) $P(2)$ iii) $P(1) + 1$ iv) $P(0) + 1$

7. The polynomial function below has a degree of 7 and has it's roots shown. Suppose all roots are single roots, how many complex roots does it have? Explain and justify your answer:



8. Factor each of the polynomials and then solve for all roots:

a) $(x^2+1)(x^2+9)=0$	b) $(x^2+10)(x^2+2x+4)=0$	c) $x^4+12x^2+35=0$
d) $(x-1+3i)(x+1+3i)=0$	e) $x^3+3x^2+x-5=0$	f) $0=x^3+3x^2+4x+12$
g) $9x^3+2x+1=0$	h) $x^4+9x^2+20=0$	i) $x^4+5x^2-24=0$

j) $x^4+x^3+7x^2+9x-18=0$	k) $8x^4+50x^3+43x^2+2x-4=0$	L) $4x^4-4x^3+13x^2-12x+3=0$

9. Given that the polynomial $f(x)=12x^5-20x^4+19x^3-6x^2-2x+1$ has roots at $x=\frac{1}{2}$ and $x=\frac{1}{3}$, find the other complex roots.

10. Given the function, how many roots are there? Find all the solutions and then indicate all NPV's if there are any.

$$\frac{(z^5+z^4+z^3+z^2+z^1+1)(z^7+1)}{z^5+1}=0$$

11. Given the polynomial $P(x)=x^3+3x^2+Bx+C$ with real coefficients and a complex root at $z=1-4i$, find the coefficients "B" and "C".

12. Given the polynomial $P(x)=x^3+Ax^2+Bx+24$ with real coefficients and a complex root at $z=3+2i$, find the coefficients "A" and "B".

13. There are nonzero integers "a", "b", "r", and "s" such that the complex number $r+si$ is a zero of the polynomial $P(x)=x^3-ax^2+bx-65$. For each possible combination of "a" and "b", let $p_{a,b}$ be the sum of the zeroes of $P(x)$. Find the sum of the $p_{a,b}$'s for all possible combinations of "a' and 'b" [2013 AIME]

14. Let "S" be the set of all polynomials of the form z^3+az^2+bz+c, where "a", "b", and "c" are integers. Find the number of polynomials in "S" such that each of its roots "z" satisfies either $|z|=20$ or $|z|=13$ [2013 AIME II]

15. Let $P(x)$ be a polynomial with integer coefficients that satisfies $P(17)=0$ and $P(24)=17$. Given that $P(n)=n+3$ has two distinct integer solutions n_1 and n_2 , find the product $n_1\times n_2$ [AIME 2005]

16. For certain real values "a", "b", "c" and "d', the equation $x^4+ax^3+bx^2+cx+d=0$ has four non-real roots. The product of two of the non real roots is $13+i$ and the sum of the other two non real roots is $3+4i$. Find the value of "b". [Hint: Since all the coefficients are real, the roots must come in conjugate pairs] Aime 1995

17. The polynomial $P(x)=\left(1+x+x^2+....x^{17}\right)^2-x^{17}$ has 34 complex roots of the form $z_k=r_k\left[\cos(2\pi a_k)+i\sin(2\pi a_k)\right],\ k=1,2,3,....34$ with $0<a_1\le a_2\le....\le a_{34}\le 1$ and $r_k>0$. Given that $a_1+a_2+a_3+a_4+a_5=\dfrac{m}{n}$, where "m' and "n" are relatively prime positive integers, find "m" and "n" [2004 AIME 1]

18. The polynomial $f(z)=az^{2018}+bz^{2017}+cz^{2016}$ has real coefficients not exceeding 2019, and $f\left(\dfrac{1+\sqrt{3}i}{2}\right)=2015+2019\sqrt{3}i$. Find the value of $f(1)$ [AIME 2019]

19. Let "P" be the product of the roots of $z^6+z^4+z^3+z^2+1=0$ that have a positive imaginary part, and suppose that $P=r(\cos\theta°+i\sin\theta°)$ where $r>0$ and $0°<\theta<360°$. Find θ [AIME 1996]